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Measuring Mathematical Anxiety: Some Thoughts from a Survey of Undergraduate Students

As reported in a previous paper, teaching mathematics to non-specialist undergraduate students in the UK has become particularly problematic in recent years. A larger scale survey of first year undergraduate students was subsequently undertaken to explore levels of mathematical anxiety among the student group. This paper illustrates and reports on the application of Rasch analysis to these survey results in order to explore the performance of the questionnaire as an instrument for measuring mathematical anxiety. The Rasch analysis enables the efficacy of questionnaire items to be explored together with the consistency of the responses recorded. Some specific weaknesses in the questionnaire items are revealed by the analysis.

Introduction

In 2008 a paper was published in *Mathitudes* describing the results of a small pilot study designed to explore the levels of mathematical anxiety experienced by students studying mathematics at first year university level (Warwick, 2008). The sample of 16 students was drawn from a larger cohort of undergraduate students all studying computing and as part of their first year curriculum all such students must study and pass a mathematics module before continuing to their second year studies.

For reasons that were discussed in that paper, the expansion of Higher Education in the UK has tended to produce first year undergraduate cohorts with widely varying levels of skill, aptitude and experience in mathematics and so the teaching of mathematics has become quite problematic. In particular, the emphasis now placed on mathematics as underpinning the study of many disciplines in an 'information rich' society requires educators to help students overcome some of the anxieties that many feel in learning mathematics.

Mathematical anxiety can be defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972) or perhaps more succinctly as "… the feelings of anxiety that some individuals experience when facing mathematical problems" (Sheffield & Hunt, 2007). Such feelings of anxiety in relation to mathematics are thought to have multiple possible causes including negative learning experiences in the classroom or at home (Rossnan, 2006), anxiety passed on from parents or teachers (Fiore, 1999), generally negative life experiences involving contact with numbers, e.g. dealing with debt, tax calculations or generally understanding bank or credit card statements (Curtain-Phillips, 2004), or general exposure to what Preis and Biggs termed 'maths myths'; statements that promote negative images of mathematics such as "women can't do math" (Preis & Biggs, 2001). Although mathematical anxiety may originate early in the mathematical lives of sufferers, many researchers believe that it is learned (Sheffield & Hunt, 2007) and could become an issue at any stage of an individual's mathematical education.

Mathematical anxiety does not just impact on students studying science, technology, engineering and mathematics (termed STEM subjects). Students taking subjects in healthcare or the social sciences for example also require skills in numeracy and an ability to manipulate numerical data or algebraic expressions. Such students may previously not have had positive learning experiences in mathematics leading to the possibility that anxiety will be induced when mathematics reappears as part of their curriculum. Furthermore, mathematical anxiety can alter the future study and career choices made by students and researchers have argued that mathematically anxious students would tend to move away from studying these STEM subjects (Chipman *et al*, 1992).

So, how can educators assist students with such anxieties? The literature suggests that, *inter alia*, this may be in the form of empowering students with self-help strategies or it may be in designing learning, teaching and assessment strategies that try to alleviate anxiety. In the former category students might be encouraged to look at their own experiences in mathematics so that they might "… get to know (a) themselves, (b) how they learn best, and (c) how to create their own success experiences." (Taylor & Brooks, 1986). In the latter category the teacher needs to try and ensure that appropriate models of learning are being used that reflect the different learning styles of students (Clute, 1984), that classes are pitched at the appropriate level (Metje *et al.*, 2007), and that there is a supportive learning environment bridging the gap between concrete learning and abstract thought (Taylor & Brooks, 1986).

What seems clear though is that improving our understanding of mathematical anxiety and the ways in which it can be measured should be a priority for mathematics educators at all levels of mathematics education.

In order to find out more about our own computing student cohort, the initial pilot study was conducted to address three broad questions. Firstly, could we use a questionnaire to learn more about the levels of mathematical anxiety experienced by these students and could such measurements be used to identify students who were at risk of failure? Secondly, to what extent could mathematical anxiety be linked with previous positive or negative educational experiences described by the students? Thirdly, could teaching of the module be enhanced in such a way as to reduce the feelings of anxiety among the students?

In relation to the first of these questions, the data gathered from that small pilot study were unable to show any clear link between anxiety and the ability of students to pass the unit. This was somewhat of a surprise as it was expected that there would be some evidence of a relationship, particularly as we did see evidence of a link between previous educational experiences and mathematical anxiety (the second research question). Good or poor previous mathematical experiences are both suggested in the literature to have an effect on engagement and mathematical self-efficacy which can, in turn, influence assessment performance.

It was felt that there were three possible reasons for this lack of evidence for a relationship:

1. there may actually be no relationship at all;
2. the pilot sample was just too small to observe a clear relationship among the 'noise' that can accompany such data on personal judgement;
3. it was possible that the mathematical anxiety questionnaire being used was just not asking questions appropriate to the sample group so that the data collected was not appropriately discriminating between students' levels of anxiety.

As reported in Warwick (2008) it was felt unlikely that there is no relationship at all since researchers have previously reported on the interaction between mathematical anxiety and the achievement of mathematics learning outcomes. Preis and Biggs (2001) describe how poor mathematical experiences lead to avoidance of mathematics with a consequent deterioration in preparation for further work, weakening performance and hence further poor experiences. A more recent study of European university students studying either business administration, economics or finance (Yenilmez *et al*., 2007) concluded that poor levels of mathematical success were associated with higher levels of mathematical anxiety and that the same was true of general academic success (other than in mathematics).

To explore the second and third suggested reasons further, a larger sample of data was collected in a subsequent academic year using the same questionnaire to measure mathematical anxiety. Again, results were correlated against student performance to uncover any relationships suggested by the data, and the questionnaire results were examined using Rasch analysis to gain some insights as to the effectiveness of the questionnaire for eliciting anxiety levels across the range of anxiety levels exhibited by the students.

The results of the data collection exercise have been reported elsewhere (Warwick, 2010) but will be summarised again below. The main focus of this paper will be the Rasch analysis of the questionnaire and the lessons that can be learned from this.

The Larger Empirical Study

At the start of the academic year in 2009 a sample of 101 students were asked to complete a questionnaire designed to determine their level of mathematical anxiety. As in the previous pilot study, a modified version of the RMARS questionnaire (Plake and Parker, 1982) was used modified to tailor the statements to the UK education environment and to update some of the terminology (for example the use of 'whiteboard' rather than 'blackboard', 'calculator' rather than 'tables in the back of the book'). The questionnaire consisted of a set of 24 statements (each briefly describing a scenario involving mathematics) and for each of these the students were asked to indicate how anxious they would feel on a 7-point Likert scale ranging from 'Not at all anxious' (denoted by 1) to 'Very anxious' (denoted by 7). Statements encompassed situations involving assessment (thinking about a mathematics test the day before), classroom learning (watching a teacher solve an equation using algebra on the whiteboard) and learning support (buying or borrowing a mathematics text book).

The compulsory mathematics module was part of the curriculum of students studying computing at two levels: Higher National Diploma (HND) and single honours degree (BSc) with the former being roughly equivalent to the first two years of study for the latter. Students are generally admitted to the HND programme with lower entry qualifications than for the BSc programme and although they can progress to BSc having completed the HND, student would be expected to take additional credits and study for an extra semester. In this way the sample of students was clustered into two groups consisting of a group of year 1 HND students (n_1 = 51) and year 1 BSc students (n_2 = 50).

The median anxiety score was calculated for each of the 24 questions for each cluster separately and the resulting analysis of the data (Warwick, 2010) confirmed that for all but one of the questionnaire items, HND students reported higher, or equal, levels of mathematical anxiety when compared to BSc students and the main sources of anxiety for both groups of students were those relating to elements of assessment.

In terms of mathematical performance, it was found that for HND students, there was a significant negative statistical correlation calculated as -0.653 (p = 0.000) between total anxiety score for a student and that student's performance in a mathematical skills assessment. This illustrated that HND students who demonstrate better skills in basic mathematics also exhibited lower anxiety scores. This relationship was not found to be significant with the BSc student group.

In summary, the larger study did indicate the type of relationship between mathematical anxiety and mathematical assessment performance that we might expect for HND students but this was not apparent for BSc students. To explore this further, we now report on an analysis of the questionnaire and its resulting data by applying the Rasch modelling approach.

The Rasch Model

As previously described, the purpose of the questionnaire was to present the student with 24 situations involving an element of mathematics learning or assessment, and to ask the student to describe the level of anxiety induced by this situation on a 7-point Likert scale. The total score for a student over all 24 situations gives an indication of the general level of mathematical anxiety exhibited by the student. The minimum total would be 24 and the maximum would be 168.

In undertaking this process we are making a number of assumptions about the questionnaire as an instrument for the measurement of mathematical anxiety. One assumption is that the Likert scale exhibits the properties of interval data over a common scale for all items with each item having equal weight; and a second is that all items on the questionnaire are measuring precisely the same thing i.e. mathematical anxiety.

The Rasch model (Rasch, 1980) is an approach to the modelling of questionnaire data that examines both of these aspects as we shall later see. Rasch analysis is a mathematical process but in this paper the mathematical formulation is eschewed in favour of a more general non-mathematical description.

There are many examples in the literature describing the use of Rasch analysis (see Bond & Fox (2008) for an excellent discussion), and one such is in the analysis of multiple-choice test results in which the analysis is able to rank the students in terms of their 'ability' and the multiple-choice questions in terms of their 'difficulty' (Edwards & Alcock, 2010). Rasch analysis tries to identify a true interval scale (measured in standard units called 'logits') on which we can place each respondent to show in our case their respective levels of anxiety (rather than ability). Also on the same scale we can show each questionnaire item (situation description) so that we can judge not the difficulty of endorsing the correct answer (as in the case of multiple-choice questions), but the difficulty of endorsing an answer as anxiety inducing. Such a placement is shown in the person map of items in Figure 2.

The common scaling adopted by Rasch analysis takes the form of a curve as shown in the Test Characteristic Curve of Figure 1.

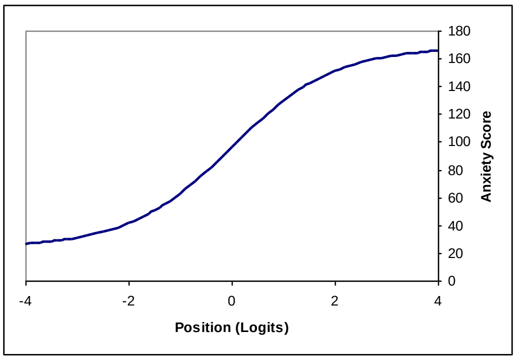

Figure 1. Test Characteristic Curve

The curve relates the position of each respondent located on the logit scale (x-axis) to the anxiety scale (y-axis). Similarly, we can plot the Item Characteristic Curve that relates questionnaire items positioned on the same logit scale with student responses via a similarly shaped curve as shown in the examples of Figures 4 and 5.

Once the analytical process has placed the respondents and questionnaire items on the common logit scale, we can immediately begin to see the extent to which the items contribute to the measurement of mathematical anxiety. But further to this, the Rasch model allows us to explore:

- the degree to which each item on the questionnaire is able to consistently distinguish between the respondents. In other words, we would expect a generally more math-anxious student to respond with higher anxiety levels across all questionnaire situations than a generally less math-anxious student.

- the degree to which each respondent is able to consistently identify the greater or lesser anxiety inducing situations.

If our questionnaire exhibits both these types of invariance then we can conclude that it is effective in assessing mathematical anxiety and we would have great confidence in using the total anxiety score as a measure of anxiety for each respondent.

On the other hand, if some items on the questionnaire seem to be inconsistent in that the student responses do not match the pattern of responses suggested by other items, then perhaps they are not measuring just anxiety, but responses are being confounded by other factors. These items can be identified and looked at again, perhaps reworded, or removed altogether from the questionnaire.

Thus our analysis will proceed in three stages. First to examine the person map of items to see what this tells us about the student sample and the questionnaire items. Second, to look at the consistency of the questionnaire items and, third, to explore the consistency of the student responses.

Rasch Analysis of the Anxiety Questionnaire

Questionnaire data for all the 101 respondents was input into the WINSTEPS Rasch analysis software as described in Bond & Fox (2008). In Figure 2, the vertical axis represents our scale in logits and each 'X' represents a student in the sample. Thus Figure 2 shows immediately that our student sample is quite tightly banded from the most anxious (scores of approximately 1 logit) to the least anxious (scores approaching -2 logits).

We would expect in Rasch analysis to see scores across the full range of possible logit values which would typically be from -4 to +4. On the vertical scale M, S and T represent the position of the mean, one and two standard deviations respectively.

In addition, if we examine the position of each questionnaire item (identified by short descriptors to the right of the vertical axis) we get an impression of how difficult it was for students to endorse each situation as anxiety inducing. Thus 'Using a calculator' was the most difficult of the situations to endorse as inducing anxiety so that, in other words, it induced the least anxiety in students. At the other extreme, 'Waiting for test results' was the situation that was the easiest to endorse as anxiety inducing. The ordering of the questionnaire items is not particularly surprising as in previous research we have found that the more passive learning activities seem to induce the least anxiety whereas those activities relating to assessment the most.

What is more surprising though is that the questionnaire items only span a small range of the vertical logit scale in Figure 2.

The implication of this is that there is not a good match between the questionnaire items and the student sample. There are students in the sample who have anxiety levels too low to be accurately measured by the questionnaire items so that they would tick most items as inducing little or no anxiety. Thus the questionnaire gives us little information about these students.

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Figure 2. Person Map of Items

Similarly, there are students who have anxiety levels that are too high to be accurately measured by this questionnaire and so we have little information on these students also. In an ideal world we would want to see the questionnaire items covering roughly the same spread of logit values as the student sample so that we could be confident that we are capturing a range of responses from all students. Our situation is analogous to a multiple choice test in which some students scored full marks while others scored 0. Our questions have not matched properly the abilities of the student group.

Turning now to the second stage of the analysis we can explore the extent to which the questionnaire items are exhibiting consistency in the way that they describe the anxiety levels of students. Figure 3 shows the logit score and MNSQ for each questionnaire item. MNSQ is a measure of the mean squared error of each item and it is recommended that an acceptable fit is given by MNSQ values within the range 0.6 to 1.4 (Bond & Fox, 2008).

Questionnaire Item	**Logit**	**MNSQ**
Using a calculator	0.26	1.06
Reading a maths book	0.20	0.60
Reading maths words	0.20	0.63
Borrowing maths book	0.19	1.31
Seeing maths on timetable	0.19	0.93
Walking to maths classes	0.19	1.09
Listening to another student	0.15	1.43
Listening to a maths lecture	0.14	0.86
Calcs on a whiteboard	0.13	1.09
Thinking about a maths unit	0.12	0.85
Starting a new maths topic	0.09	0.82
Collecting a book for coursework	0.07	0.73
Taking a maths skills test	0.07	0.92
Reading a graph	0.06	0.70
Seeing a formula in a class	0.03	0.82
Getting help with a problem	0.03	1.04
Solving an equation	-0.03	0.83
Revising for a maths test	-0.03	0.82
Taking a maths phase test	-0.04	0.80
Working on a maths problem	-0.10	0.80
Homework for next week	-0.36	1.39
Thinking about a maths test	-0.50	1.39
Given a surprise maths test	-0.51	2.07
Waiting for test results	-0.51	1.58

Figure 3. Questionnaire Item Fit

The only two items in the questionnaire that do not seem to have acceptable fit levels are those relating to 'Given a surprise maths test' and 'Waiting for test results'. We can now examine these items in more detail to try and see why the fit is poor.

To begin with let us look at an item that fits well. Figure 4 shows the fit of the data to the theoretical curve for the item 'Reading maths words'. It can be seen that there is a good fit between the empirical data and the theoretical model and all points are within their 95% confidence interval which is shown by the lines bounding the empirical data points.

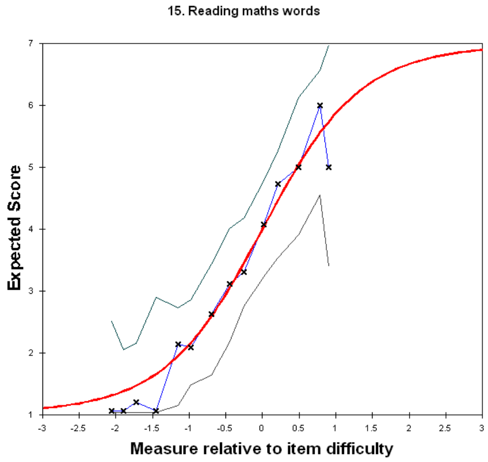

Figure 4. Questionnaire Item with Good Fit

Figure 5 on the other hand shows the fit of data for the worse fitting item 'Given a surprise maths test'. It can be seen that in this case the fit of the data to the theoretical line is poor.

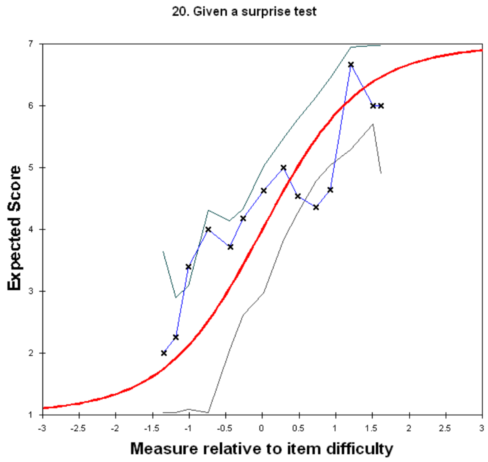

Figure 5. Questionnaire Item with Poor Fit

In Figure 5 we can see that there are data points outside the 95% confidence interval and that points to the left of the diagram are consistently above the theoretical line. This indicates that for this item, students are indicating greater levels of anxiety than we would expect and so we would need to explore this further with students to try and understand their reasoning behind the responses they have made.

Finally, the third stage of our analysis is to look at the extent to which the students in the sample have been consistent in their responses to the questionnaire. Rasch analysis allows us to look at this in a number of ways and we look at two of these here.

Firstly we can obtain an overall measure of the mean squared error as we did for the items and this will provide an overall measure of fit for each student. From the sample of 101 students in this study those six with the highest MNSQ values are shown in Figure 6.

Student	**Raw Score**	**Rasch Score**	**MNSQ**
48	133	0.80	3.03
5	37	-1.65	2.92
63	115	0.38	2.76
55	111	0.30	2.40
9	123	0.56	2.25
98	36	-1.71	2.22

Figure 6. Respondents with Poor Fit

We can then further explore this by looking at those items in the questionnaire that had the most unexpected results and perhaps go back to the students and explore the reasoning behind their responses. In Figure 7 items with the most unexpected response have been highlighted.

Student	**Expected**	**Observed**	**Item**
48	5.54	1	Seeing a formula in a class
5	1.81	7	Homework for next week
63	5.73	1	Given a surprise maths test
55	5.6	2	Waiting for test results
9	4.91	1	Calcs on a whiteboard
98	1.89	7	Waiting for test results

Figure 7. Unexpected Responses

Clearly there may be a number of practical reasons why these responses look so odd ranging from simple transcription errors to the student not reading an item carefully. However further exploration of these unexpected results would be beneficial to our understanding of the students' responses.

Summary and Conclusions

In this paper we have applied Rasch analysis in order to better understand the nature of the responses achieved in a questionnaire designed to explore mathematical anxiety in first year undergraduate computing students. The analysis allows us to draw three main conclusions.

Firstly, the set of items in the questionnaire are not appropriate to allow us to measure with confidence the full range of math anxiety in students for whom it was designed. The analysis seems to indicate that the items describe situations which can only be said to measure moderate levels of math anxiety with any degree of confidence. Extremely math anxious students seem to find all the situations anxiety inducing and confident students are not anxious about any of them. Clearly the questionnaire needs to a have spread of situations that are more inclusive of the full range of student responses.

Secondly, the items as used in the questionnaire seem to be consistent in their measurement of anxiety with the exception of two items which fit poorly to the theoretical model. Both of these items relate to assessment and we need to look at these again to see whether it is the case that other factors relating to assessment are confounding our results so that we are not measuring just mathematical anxiety.

Thirdly, the theoretical model indicates a good fit for just over 70% of the students. The remainder have a mean squared error value that is not within tolerance and we need to look at these to try and determine why the student responses are not being observed as predicted by the model.

The application of Rasch analysis has given us an opportunity to explore in some depth the results of the math anxiety questionnaire. Although the results of this larger study confirmed some findings from the pilot study, there was no significant relationship between the anxiety scores of BSc students and their mathematical performance. This might be partly explained by the fact that the majority of the poorly fitting responses were from BSc students so that their anxiety scores should not be considered with any degree of confidence.

Clearly there is more work to be done in fine-tuning this questionnaire for use with students. Further consideration of the questionnaire items is to be undertaken and a sample of students identified as being associated with unexpected responses will be further interviewed. Subsequently, another large survey will be undertaken with the next cohort of students.

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