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Using Multiple Representations to Illustrate Division by a Fraction

In this paper, the author examined the results of one classroom assignment submitted by preservice teachers to explore different representations of a division-by-fraction expression. Given one expression, the preservice teachers were asked to evaluate the expression, write a word problem modeling this expression, and describe an activity using manipulatives to illustrate the expression. The preservice teachers, the majority of whom are elementary generalists, were also asked to describe their math confidence in understanding the concept of division by a fraction.

Introduction

It is well documented that mathematics anxiety cuts across many lines, for example, gender, race, and age. There are also many factors that can bring about or heighten math anxiety, such as parental influences, negative school experiences, low math achievement, lack of confidence, and math background (Sloan, 2010). For preservice teachers this can be especially challenging, since generally they have the highest levels of math anxiety compared to other college majors (Hembree, 1990). This math anxiety can manifest itself in ways that are harmful to students. Studies have shown that preservice teachers, as well as teachers with negative attitudes toward math, tend to rely on rules and standard procedures to illustrate mathematical problem solving instead of developing comprehensive understanding of mathematical concepts and mathematical reasoning (Ball, 1990a; Karp, 1991).

Fractions and their underlying concepts present a dilemma for many teachers. Anghileri (2000) stated that viewing fractions just as parts of a whole should be avoided, and Mack (1995) found that elementary school children struggle make the conceptual jump from understanding whole number quantities to the concept that parts of a whole can be quantified in terms of the whole. Given that United States teachers have low levels of fundamental knowledge of fractions (Luo, Lo, & Leu, 2011) and that the "invert and multiply" strategy of dividing fraction could simultaneously be the most mechanical and least understood mathematical procedure in elementary mathematics curriculum (National Council of Teachers of Mathematics, 2000), preservice elementary teachers have many potential obstacles to overcome in order to prevent math anxiety regarding fractions from spreading to their potential students.

There is a common perception regarding the difficulty of fractions and their use as a mathematics tool and within the context of applications (i.e. word problems) (Brown & Quinn, 2006). Students have a difficult time working with fractions and understanding the concept of fractions (Bracey, 1996), and for preservice teachers this difficulty can bleed over into their development of necessary contextual understanding of fractions (Simon, 1993). National Council of Teachers of Mathematics (NCTM) teaching standards require that teachers who are teaching fractions be able to work with them in multiple ways, including visually through the use of manipulatives (NCTM, 2000). Son and Crespo (2009) state that, based on their findings, preservice teachers should be exposed to non-traditional mathematical strategies which can be used with division of fraction. The use of manipulatives, which can encourage the use of deeper levels of reasoning, can create an unintentional level of abstractness if not used properly. This level of abstractness and formalism is certainly not limited to fractions. This is true for many fields within math, one example being the teaching of the concept of negative integers (Altiparmak & Özdoğan, 2010). Creativity in regards to how fractions are presented and demonstrated, such as in the use of web-based instruction (Cheng-Yao, 2010) and virtual manipulatives (Ngan Hoe & Ferrucci, 2012), can help facilitate learning of the concept. Preservice teachers need to be exposed to this creativity so they can incorporate it into their mathematical content knowledge as well as the pedagogical content knowledge. If they lack an extensive, or even adequate, mathematical background, these preservice teachers may not understand how to properly implement what the teaching standards ask of them.

The process of building the desired mathematical background for a preservice teacher is not a singular task. Begg (2011) states that "teaching requires more than lecturing" (pg. 843). There is, however, a lack of foundation in regards to teaching how to teach math. According to Ball, Sleep, Boerst, and Bass (2009), there is no common curriculum designed to teach teachers how to teach math. One hypothesis suggests that there are seven categories of teacher knowledge for teachers, regardless of the subject (Shulman 1987). Knowing mathematical concepts is thus not enough; it is important to know how to work with math and understand how math can be applied in real-world applications. Fujita and Yamamoto (2011) state that a mathematically-rich task should satisfy the three principles of "offering good mathematical content, purpose, and utility" (pg. 250). These points highlight the difficulty preservice teachers face: there is no curriculum designed to teach how to teach, the knowledge teachers must possess extends beyond content, and the tasks they should use and be able to create should involve more than just content.

NCTM *Standards* stress the various aspects of contextual understanding of fractions, starting in grade three (NCTM, 2000). The *Common Core State Standards for Mathematics* aim to implement standards regarding the teaching of fractions that will help secondary students develop an intuitive feel for fractions as numbers, how to compute with fractions, and then develop a formal mathematical background that enables the use and manipulation of fractions necessary to fully utilize them (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The need to properly teach students how to work with and use fractions means teachers need to be fluent not just in the algorithmic manipulations of expressions involving fractions but they should also firmly understand the content knowledge involved with fractions. However, even if a preservice teacher has good content knowledge of mathematics, they may rely on their beliefs of mathematics as knowledge to guide their teaching practice (Beswick, Callingham, & Watson, 2012) or they may rely on a provided curriculum package (Reinke & Hoe, 2011). Even this reliance on a prepackaged curriculum is fraught with potential trouble, as the teacher may not have adequate guidance to use the curriculum package, they may have a curriculum package that does not address state or local standards that students are expected to meet, or the teacher may not have a complete set of curriculum materials to use (Kauffman, 2005). Thus it is important to consider what mathematical content knowledge preservice teachers have and how they use this knowledge.

This study aims to examine the ability of a group of preservice teachers to work with the concept of division with fractions. In addition to asking the students to perform the algebraic computational work, the preservice teachers were asked to write a word problem (real-world application) that modeled the algebraic expression and they were asked to describe how they might model the expression using physical manipulatives. Finally, the preservice teachers were asked to describe their self-efficacy regarding their understanding of the concept of division by a fraction. Of interest are the connections made by the preservice teachers within the three different representations for the division problem and how well they feel they understand the concept and not just the computational procedure.

In the following section the content knowledge necessary for teaching fractions will be discussed as will preservice teachers' self-efficacy regarding their ability to teach math. The use of manipulatives, especially with respect to fractions, will be discussed. Finally, the results will be presented as will a discussion regarding what might be addressed to help close the gap between the computation and the concept of division by a fraction.

Theoretical Background

Mathematical Content Knowledge

The mathematical content knowledge that preservice teachers possess is generally not the same as the content knowledge they should possess when they become teachers. Hill, Sleep, Lewis, and Ball (2007) state that while it is difficult to specify the knowledge required for the effective teaching of mathematics, simply possessing knowledge of mathematics is not sufficient. Teachers delivering unclear or incoherent explanations can interfere with their students' learning (Weiss & Parsley, 2004). Being able to provide explanations, though, is a fundamental teacher practice, regardless of subject, since examples and explanations help bridge the material from old concepts to new concepts as well as helping students with any misunderstanding they may have (Grossman & McDonald, 2008). Ball, Sleep, Boerst, and Bass (2009) state that "skilled mathematics teaching… requires knowing and using mathematics in ways that are distinct from simply doing math oneself" (pg. 461). Unpacking mathematics and mathematical ideas should be a skill mathematics teachers possess as well as the ability to scaffold ideas for the benefit of their students' learning (Hill & Ball, 2009). Today with the emphasis growing to better attract and prepare our young learners in the areas of Science, Technology, Engineering, and Mathematics (STEM) fields, classroom teachers need to be better equipped to reach students and instill mathematical confidence and content knowledge.

The mathematical content knowledge itself can get lost among what the preservice teachers already know. Begg (2011) states that mathematics educators tend to focus on questions with one answer instead of questions that encourage creative thinking. Indeed, there is evidence suggesting that preservice teachers will resist adoption of ideas presented to them in their preservice education (Davis, 1999). This lack of adoption of ideas presented to preservice teachers should not be surprising; even in the field of mathematics education, the issue of creativity was itself neglected (Haylock, 1987). Hill and Ball (2009) state that even simple examples illustrate "the mathematical demands of making mathematics comprehensible to students, and make clear that the mathematical knowledge involved (in problem solving) is more than being able to solve the problems oneself" (pg. 69).

Preservice teachers should understand that prior knowledge is itself insufficient for effective teaching of mathematics. In fact, the knowledge necessary for preservice teachers to have, while involving mathematical content, is different from the knowledge a mathematical researcher would have. Mathematical knowledge, for some, is viewed “as absolute and unquestionable” (Mendick, 2005, p. 247). Barton (2011) states that the prevalent attitude toward how mathematics is taught still falls in the context of providing example after example, and thus the concept of success in mathematics is equivalent to being able to find solutions to mathematical exercises. The nature of mathematics can be summarized as an ‘absolutist’ vs. ‘fallibilist’ dichotomy (Ernest, 1999); the nature of mathematics instruction can be described as a ‘content-focused’ vs. ‘learner-focused’ dichotomy with an emphasis on performance (Kuhs & Ball, 1986). It is this move toward a learner-focused environment that is shifting how preservice teachers are taught and how it is hoped they will approach teaching in their own classrooms.

Initiatives in the United States focus on improving teachers’ mathematical content knowledge, but these initiatives also include other job aspects such as teaching methods and classroom management skills (Wilson, Floden, & Ferrini-Mundy, 2002), that is, increasing the teachers’ pedagogical content knowledge (PCK). PCK can be summarized as the following knowledge base: (a) what it means to teach a particular subject; (b) instructional strategies and representations for teaching particular topics; (c) students’ understanding and potential misunderstandings of a subject area, and (d) curriculum and curricular materials (Shulman, 1986; Howey & Grossman, 1989; Grossman, 1990). Of particular interest to this study is the ability to use representations to teach a particular topic.

Representations in Mathematical Teaching

The use of representations to teach mathematical concepts is highly encouraged among teachers as a pedagogical tool. NCTM *Standards* states that “when students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically’’ (NCTM, 2000, p. 67). Indeed, students are expected to be able to use representations as a mathematical tool within their problem-solving abilities as well as to communicate mathematical ideas and model mathematical phenomena (NCTM, 2000). *Common Core State Standards for Mathematics* state that in kindergarten, students should be able to use numbers to represent quantities, to solve simple quantitative problems, and solve simple addition and subtraction scenarios by modeling them with objects (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The use of representations by teachers, though, is fraught with gaps in teachers’ ability to use representations in both teaching math and doing math (Izsa´k & Sherin 2003); Hill and Ball (2009) observed that the task of creating representations that are mathematically accurate as well as helpful to the mathematical learner is not straightforward. This presents problems for mathematical students, since “learning more means learning deeper, learning broader and seeing things differently” (Begg 2011). Students have trouble making the necessary connections between visual representations of mathematical concepts and the formal definition of said concept, according to Alcock and Weber (2010), and thus the lack of a teacher’s ability to properly use representations can only compound students’ mathematical difficulties.

There is no agreement in the literature on what a representation is. Goldin (2002) states that a “representation is a configuration that can represent something else” (p. 208). A common assumption made is that representations are physical objects; items such as cubes, cones, pattern blocks, base-ten blocks, and currency are often used to help students learn about concepts such as volume, area, place value, and addition and subtraction. These types of physical manipulatives fit under the definition Hynes provides, which is that manipulatives are “concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students” (Hynes, 1986, p. 11). Swan and Marshall (2010) give a more encompassing definition of a manipulative, saying “a mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered,” (p. 14) so that the physical item referred to as a manipulative stimulates thinking and is not just a teaching tool (e.g. calculator or fraction chart). Virtual manipulatives are purposely not included in Hynes’ definition of a manipulative.

The concept of what a representation is, though, is expanding. For example, diagrams drawn by students are cited as a recommended mathematical problem-solving heuristic (NCTM, 2000). Of wider encouraged use, though, virtual manipulatives, such as dynamic geometry software (Baki, Kosa, & Guven, 2011) and the Illuminations online activities provided by the NCTM, are simply a continuation of the use of technology to expand student assessment and instructional tools (Johnson, Campet, Gaber & Zuidema, 2012). Lee and Chen (2010) state that virtual manipulatives often are “exact visual replicas of concrete manipulatives placed on the Internet in the form of computer applets” (p. E17). In a fashion similar to Swan and Marshall, a virtual manipulative is defined by Moyer, Bolyard, and Spikell (2002) as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373).

Regardless of the type of representation used, what the definitions of manipulatives share is that they serve as a means of stimulating student engagement in mathematical learning. By creating opportunities for mathematical thinking among the students, representations, such as manipulatives, can help students develop their problem-solving skills and communicate mathematical ideas in ways that go beyond numbers, equations, and formulas.

Representations in Instruction

Using representations in mathematical instruction, no matter how beneficial for the student, can be a daunting task for the teacher. Most teachers rely on prior knowledge, using what they learned in elementary school and secondary school (Bauersfeld, 1998; Luo, Lo, & Leu, 2011). Wilson and Cooney (2002) state that a teacher’s method of instruction is shaped by his beliefs. Furthermore, Bestwick, Callingham, and Watson (2011) assert that beliefs are treated as knowledge by teachers. However, this may work in contrast to the expectations asked of a teacher. In efforts to provide deeper learning and broader learning for their students, it is believed that teachers will use multiple representations when possible in their instruction (Begg, 2011). But as Hill and Ball (2009) state, “conventional content knowledge seems to be insufficient for skillfully handling the mathematical tasks of teaching” (p. 69).

Thus asking preservice teachers to use manipulatives is, in general, asking them to use a method of instruction they may believe is transparent (i.e. the manipulative itself illuminates a desired mathematical concept) (Mitchell, Charalambous, & Hill, 2013), to use a tool they think of only as a diversion but not essential to conceptual understanding (Green, Piel, & Flowers, 2008), and, as a group, preservice elementary teachers already have a poorer attitude regarding math than the general college student body (Rech, Hartzell, & Stephens, 1993). But the use of representations, including physical and virtual manipulatives, is encouraged as part of a teacher's instructional practice (NCTM, 2000). Asking that manipulatives be used is not enough, if teachers do not have the mathematical knowledge nor the pedagogical knowledge required to use them in their lessons.

Research Questions

This study grew out of a written assignment given to a previous section of Elementary Geometry, a second semester math content course for preservice teachers. This written assignment asked students to consider a problem where pattern blocks represented numeric quantities. While students were not asked to explicitly solve the presented problem, the difficulty that students encountered in answering questions regarding what manipulatives they would use and how they would explain the concept of fractions to their future students raised questions among the students about how they conceptualized fractions.

There were two questions that drove the data analysis:

- How do preservice teachers interpret division by a fraction?
- How would preservice teachers model a word problem that involves division by a fraction?

Danielson (2010) wrote that a majority of preservice teachers, when asked to write a word problem involving division, wrote a word problem involving sharing; using the concept of division as measuring to write a word problem proved difficult. The writing assignment used to gather data for this study sought to examine the connection that preservice teachers formed between the algebraic representation of the expression, a word-problem interpretation of the expression, and a physical model of the expression using some type of manipulative.

This study relied on interpreting the responses given by the participants. While not meant to be inclusive of the conceptual understanding of all preservice teachers, the results are meant to provide insight into how preservice teachers interpret division by a fraction in three different representations. By using the data gathered, some insight can be gained by observing those connections and determining how to help preservice teachers start bridging the important connections between these different representations.

The Study

This study aimed to examine the content knowledge of preservice teachers regarding division by a fraction. The trends identified from the data evaluated, as a group, the subjects' ability to utilize a division problem where the divisor is a fraction. This utilization is approached from three points of view: simplifying a strictly algebraic problem, writing a real-word application problem using the division expression given, and providing an explanation of how the division expression could be modeled using a physical manipulative. The ability to understand the context of division by a fraction in an application problem has been included on state assessments (Texas Education Agency, 2009). Students were also asked to provide a narrative describing their personal beliefs regarding their understanding of the concept of division by a fraction. This distinction was made to counter the possibility of the subjects to answer in terms of their ability to successfully complete the algebraic division.

The Instrument

The preservice teachers were asked to complete a four-question written assignment: simplify the expression $12 \div \frac{3}{4}$, create a word problem that models this expression, describe how this expression could be modeled physically using some type of manipulative, and to write a personal reflection on how well they feel they understand the concept of division by a fraction. The preservice teachers were given over one week to complete the assignment, during which time they were allowed to consult any resource (except their peers) they deemed necessary. The assignment was created as a measure to connect material that was covered in the prerequisite course with material in the current course, as well as to examine the self-efficacy of the preservice teachers regarding the concept. The authors also desired to see any connections the preservice teachers would draw between material seen in their mathematics courses and their math methods education courses.

The Procedure

The results were coded based on the definitions of division as given by Billstein, Libeskind, and Lott (2010). This textbook was the required text for the course of interest as well as for the prerequisite course. The work on the simplification of the algebraic expression was measured with regards to the correctness of the found solution. The respective real-world applications were evaluated according to whether they satisfied any of the three provided models of division, such as the Set (Partition) Model, the Missing Factor Model, or the Repeated-Subtraction Approach. As such, word problems that modeled the equation

$$12 \div \frac{3}{4} = n$$

were coded as correct, but also word problems that modeled the equation

$$\frac{3}{4} \cdot n = 12$$

were also coded as correct, since the problem uses the Missing Factor Model.

The responses regarding the use of manipulatives were also coded with respect to the three models of division provided in the aforementioned textbook. Any relation, or lack of relation, to the respective preservice teacher's real-word application was not considered in determining if the manipulative discussion was considered valid. Thus, some responses were coded as "correct" even when they did not rely on the preservice teacher's real-world application.

Responses to the question regarding each preservice teacher's self-efficacy toward understanding the concept of division by a fraction were coded based on whether the respondent expressed confidence in their reply. This was done to distinguish between replies that affirmed the preservice teacher's ability to "flip and multiply" and express confidence at performing the division algorithm.

The Results

There were 44 students, all female, enrolled in the course, and all 44 students were required to complete the assignment. There were 36 students who submitted the assignment for grading. It was found that a majority of the preservice teachers were able to create a word problem for the expression, and the same number (although not necessarily the same students) were able to describe an activity using manipulatives to model the expression. There was, however, only a minority of the students who expressed confidence concerning their understanding of division by a fraction.

From among the 36 student submissions, there were 34 correct responses to the question asking the students to algebraically simplify the expression $12 \div \frac{3}{4}$. The two incorrect simplifications involved the same error of evaluating $12 \cdot \frac{3}{4}$ and finding a result of 9. For both of these students, they expressed a lack of confidence with some level of the material, either in working with a word problem or just the concept of division by a fraction in its entirety.

Among the submissions regarding word problems, 22 subjects provided examples that correctly modeled the division expression. Of note, 20 of the word problems submitted used the measurement interpretation of division. For example, one such word problem follows:

> "A skirt requires ¾ yards of fabric. How many skirts can be made from 12 yards?"

Only two examples illustrated a sharing/partitive division model. One example that did use a sharing model reads:

> "Tiffany has a bucket of marbles. She took three-fourths of her marbles out of the bucket and counted the marbles. There are twelve marbles out of the bucket. How many marbles total did Tiffany start with?"

There was no discussion within the classroom setting about division models of measurement nor of sharing.

From the correct submissions modeling the division expression with manipulatives, there were 22 correct examples given. For purposes of evaluation, the examples the preservice teachers provided considered correct are those that provided a tangible activity. Saying only that manipulatives would be used without specifying what manipulatives would be used or how they would be used were not considered correct. Eighteen of these models demonstrated performing the division by using the measuring method of division, and four of the examples used the sharing model. Of the four manipulative activities that were described modeling partitive division, two were submitted by students who used the measurement model in their word problem, and the remaining two partitive division manipulative activities were submitted by the students whose word problem used the partitive division model.

In expressing their confidence on understanding the concept of division by a fraction, 15 students stated they felt confident in their knowledge. Nineteen students expressed a negative view of their ability to understand division by a fraction. Two students did not address, in a positive or negative manner, their understanding of the concept of dividing by a fraction.

Discussion

There is much evidence in the literature that the education of mathematical content and pedagogy of elementary teachers needs improvement. Children have difficulty working with fractions, and the literature documents that teachers' mathematical knowledge is grounded in their K-16 education (Luo, Lo & Leu, 2011). NCTM *Standards* (2010) stresses that more needs to be done to build teachers' knowledge so that they can be mathematical thinkers. That ability to be a mathematical thinker will require teachers to be able to do more than algorithmic mathematical processes, as this is only one type of representation.

This study explored the ability of a group of preservice teachers to demonstrate their knowledge of one example of division by a fraction within three different representations. The preservice teachers were asked to first simplify the expression $12 \div \frac{3}{4}$. They were then asked to develop an application problem/word problem that modeled the given expression, and they were asked to describe how they would model the expression using physical manipulatives. Finally, the students were asked to self-assess their understanding of the concept of division by a fraction.

The results gathered show that the preservice teachers, as a group, have some difficulty creating an application that modeled the expression. Some resorted to essentially translating the expression into words. For example, one response read "Simplify the quotient of twelve and three fourths." What many of the incorrect responses offer is insight into how many of the preservice teachers tried to avoid division, as the application problem some provided relied on working with a multiplication equation involving 16 and trying to find 12 or ¾ as the result.

Among the participants, only 14 expressed confidence in their understanding of what division by a fraction is or components of such work. This is not a surprising observation (Ball, 1990b; Lo & Luo, 2012; Tobias, 2013). What is noteworthy is how low this value is, given the number of PST who correctly provided an application problem or a manipulative description or both. Only eight of the preservice teachers provided a correct application problem or described a correct manipulative activity or both yet still expressed a lack of confidence regarding their understanding of division by a fraction. The following responses illustrate the difficulty some of the preservice teachers had in describing their confidence in their understanding:

> I honestly do not feel very confident in understanding of the concept of dividing by a fraction. It's (sic) very confusing to me that when you *divide* a whole number by a *fraction* you end up with a *bigger* number. (student-added emphasis)

> I do not fully understand the concept of dividing by a fraction. I just know how to do it.

> Honestly, I understand how to do the dividing, but I don't really understand the concept. I have just always been told when dividing by a fraction to multiply by the reciprocal to find the answer. I don't really know why you multiply by the reciprocal when dividing by a fraction, it is just something I have always done.

In addition, there were also four preservice teachers who asserted their confidence in understanding the concept but did not provide a correct application problem nor a correct manipulative activity.

The disconnect that these results illustrate highlight the difficulty the preservice teachers have in connecting the process and the result. What was reaffirmed many times was the rote algorithmic method of completing the simplification of the provided expression, and this algorithmic process was mentioned throughout a number of different responses regarding their understanding of the concept. Some responses provided by the preservice teachers illustrate this:

> I don't fully understand the concept of dividing by a fraction based on what it means. I understand how to do I and ways to set it up, along with what type of answer I'm supposed to achieve.

> I understand that you can't divide by a fraction so you have to get the reciprocal of the number to make it into a multiplication problem. That is where you can simplify if needed and then multiply to get a number. The number will be bigger when you multiply by fractions.

> I know that if you are asked to divide a number by a fraction that you can flip the fraction and multiply. The number will be placed over a 1 and

> you would multiply straight across. Since I know this rule, I think I have a good grasp on how to divide by fractions.
>
> I do not understand dividing by fractions well. I don't really understand why you would get a larger number after dividing and many of my friends did not know how to do it either. I understand that it's larger because you multiply by the reciprocal but I don't understand why you would do that. I do not see the reasoning behind it, but I am willing to accept it as an equation and as a fact.

These statements show the difficulty that the preservice teachers have in understanding the underlying concept for division by a fraction even though the process for calculation is familiar. This disconnect between the different styles of representation used for the same expression shows how for some, the symbolic expression itself is the concept as opposed to being one representation of the concept. This corresponds with Stylianou (2010) and the assertion that other representations, such as those visual or application-based, are seen as methods or objects used to facilitate the completion of the mathematical work involved.

Conclusion and Summary

This research looked at the ability of preservice teachers to work with three conceptually-different representations of an expression regarding division by a fraction: an algebraic expression, the creation of a word problem/application problem, and a description of modeling this expression using a type of physical manipulative. What can be seen from this study is that there should be concern regarding the ability of preservice teachers to see that a mathematical expression is but one representation and that the ability to translate this expression into different representations is a skill worth addressing.

Green, Peil, and Flowers (2008) showed that mathematical knowledge can be improved with the use of manipulatives and that preservice teachers gained arithmetic mathematical knowledge as well as corrected mathematical misconceptions. Being able to recognize the connections between different representations can have an impact on the ability of preservice teachers to not just acquire mathematical content but also influence their ability to demonstrate and utilize their knowledge in their pedagogical practice. The low confidence that preservice teachers exhibited in this study and their math anxiety, even among those who could correctly work with the three representations asked of them, is one that can be passed on to future students (Karp, 1991).

As Vinson (2001) emphsizes, when given the opportunity to use manipulatives in practice, preservice teachers can better understand mathematical concepts and practices. Thus it would be in the best interest of preservice teachers, and their future students, to move to correct this deficiency in learning. Especially given the movement toward making mathematics more conceptual as opposed to abstract, interventions should be made to start with preservice teachers so that they take their newly-acquired knowledge and skills with them into their future classrooms to better prepare young learners to be confident in their ability to do mathematics and well equipped for a STEM world.

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