

 <b>FLORIDA ATLANTIC UNIVERSITY</b>	<b>COURSE CHANGE REQUEST</b> <b>Graduate Programs</b>	UGPC Approval _____ UFS Approval _____ SCNS Submittal _____ Confirmed _____ Banner Posted _____ Catalog _____
	Department Physics College Charles E. Schmidt College of Science	
<b>Current Course Prefix and Number</b> PHZ 5115	<b>Current Course Title</b> Mathematical Physics	
Syllabus must be attached for ANY changes to current course details. See <a href="#">Guidelines</a> . Please consult and list departments that may be affected by the changes; attach documentation. (none)		
<b>Change title to:</b> Mathematical Physics 1  <b>Change prefix</b> From:                      To:  <b>Change course number</b> From:                      To:  <b>Change credits*</b> From:                      To:  <b>Change grading</b> From:                      To:  <small>*Review <a href="#">Provost Memorandum</a></small>	<b>Change description to:</b>  <b>Change prerequisites/minimum grades to:</b> (none)  <b>Change corequisites to:</b>  <b>Change registration controls to:</b>  Please list existing and new pre/corequisites, specify AND or OR and include minimum passing grade.	
<b>Effective Term/Year for Changes:</b> Fall 2019	<b>Terminate course? Effective Term/Year for Termination:</b>	
<b>Faculty Contact/Email/Phone</b> Chris Beetle <cbeetle@fau.edu> 7-4612		
<b>Approved by</b> Department Chair _____ College Curriculum Chair _____ College Dean _____ UGPC Chair _____ UGC Chair _____ Graduate College Dean _____ UFS President _____ Provost _____	<b>Date</b> 3/12/19 3/12/19 3/27/2019 3/27/19 3/27/2019	

Email this form and syllabus to [UGPC@fau.edu](mailto:UGPC@fau.edu) one week before the UGPC meeting.

GRADUATE COLLEGE

MAR 12 2019

Received

Physics (PHZ) 5115  
 3 credit hours  
 2:00 – 3:20 TΘ  
 Science & Engineering 319A

The theory of functional analysis has become increasingly important to theoretical physics over the past century. Unfortunately, many contemporary mathematical treatments of this subject obscure its conceptual foundations and connection to practical problems of the sort that physicists routinely encounter. This is a pity both because the mathematics offers considerable insight into the structure of physical problems, and conversely because physical intuition can help shed light on why abstract mathematical ideas are defined as they are, and how they relate to one another.

This course aims to bridge the gap between the mathematical theory and its physical applications. We will study functional analysis as a means of gaining insight into practical problems in theoretical physics. Much of the course will center on the analysis of linear partial differential equations. Most students in the course probably have solved such equations as undergraduates. However, this course is not concerned solely with solving equations, but also with techniques that allow one to discern whether solutions exist, and how many, *before* actually attempting to find one. These techniques help to guide one's thinking when solving practical problems either analytically or numerically.

### Prerequisite Background

Students in this course are likely to have completed their undergraduate work at other universities. Rather than give a list specific prerequisite courses for this one, it is therefore better just to state that the plan for the course assumes that students will already have experience with the following:

- linear algebra and eigenvalue problems,
- solution of ordinary differential equations,
- Fourier series and transforms, the Dirac delta function, Hilbert spaces,
- elementary complex variables (*i.e.*, analytic functions and the calculus of residues), and
- separation of variables for the Laplacian in 2 and 3 dimensions.

Although we will review some of this material, students may struggle toward the end of the course without adequate prior experience in these areas. Any student who is concerned about his or her mathematical preparation should meet with me early in the semester to discuss the matter.



Course Web Site

<https://canvas.fau.edu/courses/52955>

#### Instructor

Prof. Chris Beetle  
 Science & Engineering 448  
 (o) 561.297.4612  
 (m) 561.306.3234  
[cbeetle@fau.edu](mailto:cbeetle@fau.edu)

Office hours: by appointment.

#### Required Text

- M. Stone and P. Goldbart. *Mathematics for Physics: A Guided Tour for Graduate Students.* (Cambridge, 2009.) ISBN 0-521-85403-0

GRADUATE COLLEGE

MAR 25 2019

Received

## Course Description and Objectives

The topics to be covered in this course have been selected, among the many that could (or perhaps even should) be covered, to focus on those that are useful in *multiple* other courses in the graduate Physics curriculum. These subjects are of course also essential in contemporary research, primarily in theoretical physics. They include

- the calculus of variations with applications,
- integral transforms and generalized functions,
- Sturm–Liouville theory and special functions,
- boundary-value problems and Green functions,
- the partial differential equations of mathematical physics, and
- integral equations and their applications.

The course will follow a traditional lecture format, with an emphasis on solving specific problems. Computer demonstrations will be used if and when possible.

## Course Schedule

The following is a rough schedule of the major topics to be covered, and the rough amount of time to be devoted to each:

- Review of Linear Algebra (1 week)
- Calculus of Variations (1½ weeks)
- Hilbert Space and Distribution Theory (1½ weeks)
- Ordinary Differential Equations (1½ weeks)
- The Green Function Method (1½ weeks)
- Partial Differential Equations I (1½ weeks)
- Partial Differential Equations II (1½ weeks)
- Legendre and Bessel Functions (1½ weeks)
- Integral Transforms and Equations (2 weeks)

Each of these will be the subject of one problem set. These will be due roughly every ten days. A more detailed schedule is attached. This schedule is fairly ambitious, however, and may change based on the pace we are able to maintain in the course.

## Evaluation

Grades in this course will be calculated using a weighted curve based on several factors.:

- 15% – written lecture notes.

One student volunteer will serve as a recording secretary during each lecture. That student will scan his or her notes and distribute them to all of the other students through the course's Canvas site. The goal of this approach is to free the rest of the students to engage more actively and ask questions. I will grade each day's lecture notes for completeness and legibility. Each student should plan to serve as secretary roughly three times during the course of the semester.

- 20% – homework assignments

Homework will be due roughly every ten days throughout the semester. I will grade each student's solutions based on completeness and organization. I will not grade the individual problems in detail because I have previously distributed my own solutions to many of the problems. Students are allowed to discuss and collaborate on the homework problems, but no copying, either from other students or from previously published solutions of the problems, is allowed.

- 45% – three written, take-home, midterm exams.

Collaboration on exams is not allowed.

- 20% – one written, in-class, final exam.

The final exam will be held in the regular classroom during the period scheduled by the University: **Tuesday, December 11 from 1:15 to 3:45**. It will cover *all* material from the course. Students may consult the textbook and their course notes, but no other resources, during the exam period.

## Course Policies

### Late Assignments

All assigned coursework must be ready at the beginning of the lecture on the date it is due. If a student has a family emergency or illness, he or she may request an individual extension *by email at least twenty-four hours before* the assignment is due. I reserve the right to approve or deny such requests. Exceptions can also be made for students' participation in University-approved activities and religious observances. Please advise me of any such conflicts.

### Extra Credit

No extra credit will be offered.

### Collaboration on Assigned Coursework

Students are encouraged to collaborate on homework assignments. Copying is not allowed. Collaboration on the exams is also not allowed.

### Incomplete Grades

Grades of Incomplete (I) are reserved for students who are passing the course, but unable to complete all assigned course work on time due to exceptional circumstances such as those outlined above for late assignments. Unless such circumstances can be documented, work not completed at the end of the semester will be assessed as a zero in the final grades.

## Important Dates

- Thursday, August 30 – Class canceled
- Friday, August 31 – Make-up lecture (regular classroom at 11:00 – 12:20)
- Tuesday, September 11 – Midterm Exam I available in class
- Thursday, September 13 – Midterm Exam I due in class

- Thursday, September 27 – Class canceled
- Friday, September 28 – Make-up lecture (regular classroom at 11:00 – 12:20)
- Thursday, October 4 – Midterm Exam II available in class
- Tuesday, October 9 – Midterm Exam II due in class
- Thursday, October 25 – Class canceled
- Friday, October 26 – Make-up lecture (regular classroom at 11:00 – 12:20)
- Thursday, November 8 – Midterm Exam III available in class
- Tuesday, November 13 – Midterm Exam III due in class
- Thursday, November 22 – No class due to Thanksgiving Recess
- Thursday, November 29 – Class canceled
- Friday, November 30 – Make-up lecture (regular classroom at 11:00 – 12:20)
- Tuesday, December 4 – Optional review session (regular classroom and time)
- Tuesday, December 11 – In-class final exam (regular classroom at 1:15 – 3:45)

#### **Attendance Policy Statement**

Students are expected to attend all of their scheduled University classes and to satisfy all academic objectives as outlined by the instructor. The effect of absences upon grades is determined by the instructor, and the University reserves the right to deal at any time with individual cases of non-attendance.

Students are responsible for arranging to make up work missed because of legitimate class absence, such as illness, family emergencies, military obligation, court-imposed legal obligations or participation in University-approved activities. Examples of University-approved reasons for absences include participating on an athletic or scholastic team, musical and theatrical performances and debate activities. It is the student's responsibility to give the instructor notice prior to any anticipated absences and within a reasonable amount of time after an unanticipated absence, ordinarily by the next scheduled class meeting. Instructors must allow each student who is absent for a University-approved reason the opportunity to make up work missed without any reduction in the student's final course grade as a direct result of such absence.

#### **Disability Policy Statement**

In compliance with the Americans with Disabilities Act Amendments Act (ADAAA), students who require reasonable accommodations due to a disability to properly execute coursework must register with Student Accessibility Services (SAS) and follow all SAS procedures. SAS has offices across three of FAU's campuses – Boca Raton, Davie and Jupiter – however disability services are available for students on all campuses. For more information, please visit the SAS website at [www.fau.edu/sas/](http://www.fau.edu/sas/).

#### **Counseling and Psychological Services (CAPS) Center**

Life as a university student can be challenging physically, mentally and emotionally. Students who find stress negatively affecting their ability to achieve academic or personal goals may wish to consider utilizing FAU's Counseling and Psychological Services (CAPS) Center. CAPS provides FAU students a range of services – individual counseling, support meetings, and psychiatric services, to name a few – offered to help improve and maintain emotional well-being. For more information, go to <http://www.fau.edu/counseling/>.

#### **Code of Academic Integrity Policy Statement**

Students at Florida Atlantic University are expected to maintain the highest ethical standards. Academic dishonesty is considered a serious breach of these ethical standards, because it interferes with the university mission to provide a high quality education in which no student enjoys an unfair advantage over any other. Academic dishonesty is also destructive of the university community, which is grounded in a system of mutual trust and places high value on personal integrity and individual responsibility. Harsh penalties are associated with academic dishonesty. For more information, see [University Regulation 4.001](#).

<u>Date</u>	<u>Lecture Topic</u>	<u>Reading</u>
	▼ <b>Mathematical Physics (PHZ 5115)</b>	
Tue 21 Aug	▼ <b>Linear Algebra: Theory</b> <ul style="list-style-type: none"> <li>• Vectors, bases and components</li> <li>• Linear maps and dual vectors</li> <li>• Inner products and adjoint operators</li> <li>• Direct sums and quotients</li> </ul>	744 – 756
Thu 23 Aug	▼ <b>Linear Algebra: Applications</b> <ul style="list-style-type: none"> <li>• Linear systems of equations</li> <li>• Matrices and determinants</li> <li>• Eigenvalues and diagonalization</li> <li>• Jordan normal form</li> </ul>	757 – 772
Tue 28 Aug	▼ <b>The Calculus of Variations</b> <ul style="list-style-type: none"> <li>• Functionals and their variations</li> <li>• The Euler–Lagrange equations</li> <li>• Lagrangian mechanics</li> </ul>	1 – 17
<i>Fri 31 Aug</i>	▼ <b>Fields and Continuum Mechanics</b> <ul style="list-style-type: none"> <li>• <i>Problem Set I Due</i></li> <li>• Many degrees of freedom</li> <li>• Continuum limit and mechanics of media</li> <li>• Fluid mechanics</li> </ul>	17 – 26
Tue 4 Sep	▼ <b>Advanced Topics in Variational Calculus</b> <ul style="list-style-type: none"> <li>• Problems with variable endpoints</li> <li>• Constraints and Lagrange multipliers</li> <li>• The second variation</li> </ul>	27 – 38
Thu 6 Sep	▼ <b>Function Spaces</b> <ul style="list-style-type: none"> <li>• Functions as vectors</li> <li>• Convergence and Hilbert space</li> <li>• Completeness and Hilbert bases</li> <li>• Best approximation and Parseval's theorem</li> </ul>	50 – 62
Tue 11 Sep	▼ <b>Review Session</b> <ul style="list-style-type: none"> <li>• <i>Problem Set II Due</i></li> <li>• <i>Midterm Exam I Available (Sets 1 &amp; 2)</i></li> </ul>	
Thu 13 Sep	▼ <b>Fourier Series and Transforms</b> <ul style="list-style-type: none"> <li>• <i>Midterm Exam I Due</i></li> <li>• Fourier series and their limits</li> <li>• Fourier transforms</li> <li>• Gibbs' phenomenon</li> <li>• The Poisson summation formula</li> </ul>	779 – 795

<u>Date</u>	<u>Lecture Topic</u>	<u>Reading</u>
Tue 18 Sep	▼ <b>Linear Operators and Distributions</b> <ul style="list-style-type: none"> <li>• Orthogonal polynomials</li> <li>• Linear operators</li> <li>• Test functions and distributions</li> <li>• Calculus with distributions</li> </ul>	62 – 75
Thu 20 Sep	▼ <b>Linear Ordinary Differential Equations</b> <ul style="list-style-type: none"> <li>• Existence and uniqueness of solutions</li> <li>• Linear independence and the Wronskian</li> <li>• Normal form and singular points</li> <li>• Solution of inhomogeneous equations</li> </ul>	86 – 98
Tue 25 Sep	▼ <b>Linear Ordinary Differential Operators</b> <ul style="list-style-type: none"> <li>• <i>Problem Set III Due</i></li> <li>• Operators, domains and boundary conditions</li> <li>• Adjoint operators and boundary conditions</li> <li>• Self-adjoint problems and extensions</li> <li>• Introduction to the eigenvalue problem</li> </ul>	101 – 116
<i>Fri 28 Sep</i>	▼ <b>Completeness of Eigenfunctions</b> <ul style="list-style-type: none"> <li>• Operators with discrete spectrum</li> <li>• Rayleigh–Ritz and other methods</li> <li>• Operators with continuous spectrum</li> <li>• Generalized eigenfunctions</li> </ul>	117 – 131
Tue 2 Oct	▼ <b>Introduction to Green Functions</b> <ul style="list-style-type: none"> <li>• The Fredholm alternative</li> <li>• Theory and methods of Green functions</li> <li>• Two-point and initial-value problems</li> <li>• The modified Green function</li> </ul>	140 – 150
Thu 4 Oct	▼ <b>Review Session</b> <ul style="list-style-type: none"> <li>• <i>Problem Set IV Due</i></li> <li>• <i>Midterm Exam II Available (Sets III &amp; IV)</i></li> </ul>	
Tue 9 Oct	▼ <b>Applications of Green Functions</b> <ul style="list-style-type: none"> <li>• <i>Midterm Exam II Due</i></li> <li>• Hermiticity and Lagrange's identity</li> <li>• Eigenfunction expansions</li> <li>• Inhomogeneous boundary conditions</li> <li>• Causality and analyticity</li> </ul>	150 – 159
Thu 11 Oct	▼ <b>Analytic Properties of Green Functions</b> <ul style="list-style-type: none"> <li>• Causality and analyticity revisited</li> <li>• Plemelj formulae and principal values</li> <li>• Resolvent operators and Green functions</li> <li>• Locality and Green functions</li> </ul>	155 – 167

<u>Date</u>	<u>Lecture Topic</u>	<u>Reading</u>
Tue 16 Oct	▼ <b>Introduction to Partial Differential Equations</b> <ul style="list-style-type: none"> <li>• Classification of partial differential equations</li> <li>• Characteristics and Cauchy data</li> <li>• First-order equations</li> <li>• The wave equation in two dimensions</li> </ul>	174 – 185
Thu 18 Oct	▼ <b>The Wave Equation</b> <ul style="list-style-type: none"> <li>• <i>Problem Set V Due</i></li> <li>• The d'Alambert and Fourier solutions</li> <li>• The retarded Green function</li> <li>• Waves in odd vs. even dimensions</li> <li>• Huygens' principle</li> </ul>	181 – 195
Tue 23 Oct	▼ <b>The Heat Equation</b> <ul style="list-style-type: none"> <li>• The heat kernel</li> <li>• The causal Green function</li> <li>• Duhamel's principle</li> <li>• The Schrödinger equation</li> </ul>	196 – 201
<i>Fri 26 Oct</i>	▼ <b>The Laplace Equation</b> <ul style="list-style-type: none"> <li>• The Poisson and Laplace equations</li> <li>• Dirichlet and Neumann problems</li> <li>• Existence and uniqueness of solutions</li> <li>• Separation of variables</li> </ul>	201 – 213
Tue 30 Oct	▼ <b>The Poisson and Helmholtz Equations</b> <ul style="list-style-type: none"> <li>• <i>Problem Set VI Due</i></li> <li>• Eigenfunction expansions and Green functions</li> <li>• Boundary value problems</li> <li>• Method of images</li> <li>• Monochromatic waves</li> </ul>	213 – 223
Thu 1 Nov	▼ <b>Dispersion and Resonance</b> <ul style="list-style-type: none"> <li>• Dispersive waves</li> <li>• Phase vs. group velocity</li> <li>• Wakes and rays</li> <li>• Rayleigh's equation</li> </ul>	231 – 246
Tue 6 Nov	▼ <b>Spherical Harmonics</b> <ul style="list-style-type: none"> <li>• Calculus in curvilinear coordinates</li> <li>• Separation of variables in spherical coordinates</li> <li>• Legendre polynomials</li> <li>• General spherical harmonics</li> </ul>	264 – 278
Thu 8 Nov	▼ <b>Review Session</b> <ul style="list-style-type: none"> <li>• <i>Problem Set VII Due</i></li> <li>• <i>Midterm Exam III available (Sets IV – VII)</i></li> </ul>	



<u>Date</u>	<u>Lecture Topic</u>	<u>Reading</u>
Tue 13 Nov	▼ <b>Cylindrical Bessel Functions</b> <ul style="list-style-type: none"> <li>• <i>Midterm Exam III due</i></li> <li>• Bessel's equation and its solutions</li> <li>• Recursion relations and other identities</li> <li>• Orthogonality and Hankel transforms</li> <li>• Modified Bessel functions</li> </ul>	278 – 293
Thu 15 Nov	▼ <b>Spherical Bessel Functions</b> <ul style="list-style-type: none"> <li>• The spherical Bessel equation</li> <li>• Recursion relations and other identities</li> <li>• Singular endpoints and regularity conditions</li> <li>• Weyl's theorem</li> </ul>	294 – 305
Tue 20 Nov	▼ <b>Integral Transforms</b> <ul style="list-style-type: none"> <li>• Introduction to integral equations</li> <li>• Fourier transforms</li> <li>• Laplace transforms</li> <li>• Radon transforms</li> </ul>	311 – 321
Thu 22 Nov	• (No Class due to Thanksgiving Recess)	
Tue 27 Nov	▼ <b>Exact Solution of Integral Equations</b> <ul style="list-style-type: none"> <li>• <i>Problem Set VIII Due</i></li> <li>• Separable kernels and the eigenvalue problem</li> <li>• Inhomogeneous problems</li> <li>• Singular integral equations and principal parts</li> <li>• Wiener–Hopf equations</li> </ul>	321 – 332
<i>Fri 30 Nov</i>	▼ <b>Approximate Methods for Integral Equations</b> <ul style="list-style-type: none"> <li>• Integral equations and functional analysis</li> <li>• Geometry of operators in Hilbert space</li> <li>• The Born approximation</li> <li>• The Fredholm series</li> </ul>	332 – 342
Tue 4 Dec	• <b>Optional Review Session (Reading Day)</b>	
Fri 7 Dec	▼ <b>“A date which will live in infamy ...”</b> <ul style="list-style-type: none"> <li>• <i>Problem Set IX Due</i></li> </ul>	
Tue 11 Dec	▼ <b>Final Exam Period (1:15 – 3:45)</b> <ul style="list-style-type: none"> <li>• <i>The final exam will be held in class.</i></li> </ul>	